

Democratic Mass Matrices from Broken $O(3)_L \times O(3)_R$ Flavor Symmetry

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Abstract

We impose $O(3)_L \times O(3)_R$ flavor symmetry in the supersymmetric standard model. Three lepton doublets ℓ_i transform as an $O(3)_L$ triplet and three charged leptons \bar{e}_i as an $O(3)_R$ triplet, while Higgs doublets H and \bar{H} are $O(3)_L \times O(3)_R$ singlets. We discuss a flavor $O(3)_L \times O(3)_R$ breaking mechanism that leads to "successful" phenomenological mass matrices, so-called "democratic" ones, in which the large $\nu_\mu - \nu_\tau$ mixing is naturally obtained. Three neutrinos have nearly degenerate masses of order 0.1eV which may be accesible to future double β -decay experiments. We extend our approach to the quark sector and show that it is well consistent with the observed quark mass hierarchies and the CKM matrix elements. However, the large mass of the top quark requires a relatively large coupling constant.

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Yukawa coupling matrices of Higgs field (i.e. masses and mixings of quarks and leptons) are the least understood part of the standard electroweak gauge theory, which are, however, expected to be an important hint of more fundamental theory. There have been a number of attempts to understand mass matrices of quarks and leptons by postulating some broken flavor symmetries based on Abelian [1] or non-Abelian [2] groups. The $O(3)$ flavor symmetry [3, 4] has a unique prediction, that is almost degenerate neutrino masses. Using the result of the atmospheric neutrino oscillation observed in SuperKamiokande experiments [5], one may conclude that all three neutrinos have masses of order $0.1 - 1\text{eV}$ for the case of degenerate neutrinos. This is very interesting since such degenerate neutrino masses lie in the region accessible to future double- β decay experiments [6] if the neutrinos are Majorana particles.

On the contrary, masses of quarks and charged leptons vanish in the $O(3)$ symmetric limit. Therefore, mass matrices of quarks and leptons are determined by details of breaking pattern of the flavor symmetry. In this letter, we discuss a possible flavor $O(3)$ breaking mechanism that leads to "successful" phenomenological mass matrices, so-called "democratic" ones [7, 8], in which the large $\nu_\mu - \nu_\tau$ mixing suggested from the atmospheric neutrino oscillation [5] is naturally obtained.

We consider the supersymmetric standard model and impose $O(3)_L \times O(3)_R$ flavor symmetry. Three lepton doublets $\ell_i (i = 1 - 3)$ transform as an $O(3)_L$ triplet and three charged leptons $\bar{e}_i (i = 1 - 3)$ as an $O(3)_R$ triplet, while Higgs doublets H and \bar{H} are $O(3)_L \times O(3)_R$ singlets. We will discuss the quark sector later.

We introduce, to break the flavor symmetry, pair of fields $\Sigma_L^{(i)} (i = 1, 2)$ and $\Sigma_R^{(i)} (i = 1, 2)$ which transform as symmetric traceless tensor **5**'s of $O(3)_L$ and $O(3)_R$, respectively.

We assume that the $\Sigma_L^{(i)}(\mathbf{5}, \mathbf{1})$ and $\Sigma_R^{(i)}(\mathbf{1}, \mathbf{5})$ take values ²

$$\Sigma_{L,R}^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} w_{L,R}^{(1)}, \quad (1)$$

and

$$\Sigma_{L,R}^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} w_{L,R}^{(2)}. \quad (2)$$

We consider that these are explicit breakings of $O(3)_L \times O(3)_R$ rather than vacuum-expectation values of $\Sigma_{L,R}^{(i)}$ (spontaneous breaking), otherwise we have unwanted massless Nambu-Goldstone multiplets. In the following discussion we use dimensionless breaking parameters $\sigma_L^{(i)}$ and $\sigma_R^{(i)}$, which are defined as

$$\sigma_{L,R}^{(1)} \equiv \frac{\Sigma_{L,R}^{(1)}}{M_f} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \delta_{L,R}, \quad (3)$$

and

$$\sigma_{L,R}^{(2)} \equiv \frac{\Sigma_{L,R}^{(2)}}{M_f} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \epsilon_{L,R}. \quad (4)$$

Here, M_f is the large flavor mass scale, $\delta_{L,R} = w_{L,R}^{(1)}/M_f$ and $\epsilon_{L,R} = w_{L,R}^{(2)}/M_f$. We assume $\delta_{L,R}, \epsilon_{L,R} \leq 1$.

The neutrinos acquire small Majorana masses from a superpotential, ³

$$W = \frac{H^2}{M} \ell (\mathbf{1} + \alpha_{(i)} \sigma_L^{(i)}) \ell, \quad (5)$$

which yields a neutrino mass matrix as

$$\widehat{m}_\nu = \frac{\langle H \rangle^2}{M} \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \alpha_{(1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \delta_L + \alpha_{(2)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \epsilon_L \right\}. \quad (6)$$

Here, $\alpha_{(i)}$ are $\mathcal{O}(1)$ parameters and the mass M denotes a cut-off scale of the present model which may be different from the flavor scale M_f . We take $M \simeq 10^{14-15} \text{GeV}$ to obtain

²The more general case for $\Sigma_L^{(i)}$ and $\Sigma_R^{(i)}$ will be discussed in the end of this letter.

³This superpotential is induced by integrating some massive heavy fields $N_i (i = 1-3)$ which transform as a triplet of $O(3)_L$ (rather than $O(3)_R$). In this case the mass M corresponds to Majorana masses of N_i .

$m_{\nu_i} \simeq 0.1 - 1\text{eV}$ indicated from the atmospheric neutrino oscillation [5] for degenerate neutrinos.

The above breaking is, however, incomplete, since the charged leptons remain massless. We introduce an $O(3)_L$ -triplet and an $O(3)_R$ -triplet fields $\phi_L(\mathbf{3}, \mathbf{1})$ and $\phi_R(\mathbf{1}, \mathbf{3})$ to produce masses of the charged leptons. The vacuum expectation values of ϕ_L and ϕ_R are determined by the following superpotential; ⁴

$$\begin{aligned} W = & Z_L(\phi_L^2 - 3v_L^2) + Z_R(\phi_R^2 - 3v_R^2) \\ & + X_L(a_{(i)}\phi_L\sigma_L^{(i)}\phi_L) + X_R(a'_{(i)}\phi_R\sigma_R^{(i)}\phi_R) \\ & + Y_L(b_{(i)}\phi_L\sigma_L^{(i)}\phi_L) + Y_R(b'_{(i)}\phi_R\sigma_R^{(i)}\phi_R) . \end{aligned} \quad (7)$$

Here, the fields $Z_{L,R}$, $X_{L,R}$ and $Y_{L,R}$ are all singlets of $O(3)_L \times O(3)_R$. The parameters v_L and v_R can be of order the gravitational scale in principle, since there is no symmetry to suppress them. We should, however, assume that they are of order the flavor scale M_f to obtain $\mathcal{O}(1)$ effective Yukawa couplings of Higgs H and \overline{H} .

We obtain vacuum-expectation values from the superpotential eq.(7),

$$\langle \phi_L \rangle \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} v_L , \quad \langle \phi_R \rangle \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} v_R . \quad (8)$$

Notice that only with the first two terms in eq.(7) we have $O(3)_L \times O(3)_R$ global symmetry and hence unwanted Nambu-Goldstone multiplets appear in broken vacua. The couplings to the explicit breakings $\sigma_{L,R}^{(i)}$ are necessary to eliminate the Nambu-Goldstone multiplets in the low energy spectrum, which determine vacuum-expectation values of ϕ_L and ϕ_R as in eq.(8). Here, we should quote a work by Barbieri et al. [4], who have also proposed a similar vacuum-misalignment mechanism.

With the non-vanishing $\langle \phi_L \rangle$ and $\langle \phi_R \rangle$ in eq.(8), the Dirac masses of charged

⁴This superpotential is consistent with R-symmetry $U(1)_R$, in which $Z_{L,R}$, $X_{L,R}$, $Y_{L,R}$, $\phi_{L,R}$ and $\sigma_{L,R}^{(i)}$ have $U(1)_R$ charges, 2, 2, 2, 0 and 0.

leptons arise from a superpotential,

$$W = \frac{\kappa_E}{M_f^2} (\bar{e}\phi_R)(\phi_L\ell)\overline{H}. \quad (9)$$

This produces so-called "democratic" mass matrix of the charged leptons,

$$\widehat{m}_E = \kappa_E \left(\frac{v_L v_R}{M_f^2} \right) \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} < \overline{H} >. \quad (10)$$

Diagonalization of this mass matrix yields large lepton mixings as shown in ref.[8] and one non-vanishing eigenvalue, m_τ . The masses of e and μ are derived from distortion of the "democratic" form of mass matrix in eq.(10), which is given by a superpotential containing the explicit $O(3)_L \times O(3)_R$ breaking parameters $\sigma_{L,R}^{(i)}$,⁵

$$\delta W = \frac{\kappa_E}{M_f^2} \left\{ A_i^\ell (\bar{e}\sigma_R^{(i)}\phi_R)(\phi_L\ell) + B_i^\ell (\bar{e}\phi_R)(\phi_L\sigma_L^{(i)}\ell) + C_{ij}^\ell (\bar{e}\sigma_R^{(i)}\phi_R)(\phi_L\sigma_L^{(j)}\ell) \right\} \overline{H}. \quad (11)$$

Then, the charged lepton mass matrix is given by

$$\begin{aligned} \widehat{m}_E = \kappa_E \left(\frac{v_L v_R}{M_f^2} \right) < \overline{H} > \left\{ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + A_1^\ell \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix} \delta_R + B_1^\ell \begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ 1 & 1 & -2 \end{pmatrix} \delta_L \right. \\ + C_{11}^\ell \begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 4 \end{pmatrix} \delta_L \delta_R + A_2^\ell \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \epsilon_R + B_2^\ell \begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ 1 & -1 & 0 \end{pmatrix} \epsilon_L \\ \left. + C_{12}^\ell \begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ -2 & 2 & 0 \end{pmatrix} \delta_R \epsilon_L + C_{21}^\ell \begin{pmatrix} 1 & 1 & -2 \\ -1 & -1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \delta_L \epsilon_R + C_{22}^\ell \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \epsilon_L \epsilon_R \right\}. \quad (12) \end{aligned}$$

The mass eigenvalues of this lepton mass matrix are

$$m_\tau \simeq 3\kappa_E \frac{v_L}{M_f} \frac{v_R}{M_f} < \overline{H} >, \quad \frac{m_\mu}{m_\tau} \simeq \mathcal{O}(\delta_L \delta_R), \quad \frac{m_e}{m_\tau} \simeq \mathcal{O}(\epsilon_L \epsilon_R), \quad (13)$$

where we assume that all coupling parameters A_i^ℓ , B_i^ℓ and $C_{ij}^\ell (i, j = 1, 2)$ are of $\mathcal{O}(1)$.

Since the analysis on the quark mass matrices below shows preferable values $\delta_R \simeq 1$ and

$\epsilon_R \simeq 0.1 - 1$, we take $\delta_L \simeq 0.1$ and $\epsilon_L \simeq 10^{-3} - 10^{-2}$.

⁵We may also have such terms as $D_{ij}^\ell (\bar{e}\sigma_R^{(i)}\sigma_R^{(j)}\phi_R)(\phi_L\ell)$ or $E_{ij}^\ell (\bar{e}\phi_R)(\phi_L\sigma_L^{(i)}\sigma_L^{(j)}\ell)$ in the curly bracket of eq.(11). These terms and higher terms of σ fields can be, however, absorbed in terms in eq.(11). In this process, the coupling parameters A_i^ℓ , B_i^ℓ and C_{ij}^ℓ remain of $\mathcal{O}(1)$.

We have an additional contribution to the neutrino mass matrix in eq.(6) as

$$\delta W = \frac{H^2}{M} \ell \left(\beta \frac{\phi_L}{M_f} \frac{\phi_L}{M_f} \right) \ell. \quad (14)$$

The neutrino mass matrix is now given by

$$\begin{aligned} \widehat{m}_\nu = & \frac{\langle H \rangle^2}{M} \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \alpha_{(1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \delta_L + \alpha_{(2)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \epsilon_L \right. \\ & \left. + \beta \left(\frac{v_L}{M_f} \right)^2 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \right\}. \end{aligned} \quad (15)$$

To see the neutrino mixing, we take the hierarchical base by applying an orthogonal transformation: the charged lepton and neutrino mass matrices are written as

$$F^T \widehat{m}_E F = \kappa_E \left(\frac{v_L v_R}{M_f^2} \right) \langle \overline{H} \rangle \begin{pmatrix} 2C_{22}^\ell \epsilon_L \epsilon_R & 2\sqrt{3}C_{21}^\ell \epsilon_R \delta_L & \sqrt{6}A_2^\ell \epsilon_R \\ 2\sqrt{3}C_{12}^\ell \epsilon_L \delta_R & 6C_{11}^\ell \delta_L \delta_R & 3\sqrt{2}A_1^\ell \delta_R \\ \sqrt{6}B_2^\ell \epsilon_L & 3\sqrt{2}B_1^\ell \delta_L & 3 \end{pmatrix}_{RL}, \quad (16)$$

and

$$F^T \widehat{m}_\nu F = \frac{\langle H \rangle^2}{M} \begin{pmatrix} 1 + \alpha_{(1)} \delta_L & \frac{1}{\sqrt{3}} \alpha_{(2)} \epsilon_L & \sqrt{\frac{2}{3}} \alpha_{(2)} \epsilon_L \\ \frac{1}{\sqrt{3}} \alpha_{(2)} \epsilon_L & 1 - \alpha_{(1)} \delta_L & \sqrt{2} \alpha_{(1)} \delta_L \\ \sqrt{\frac{2}{3}} \alpha_{(2)} \epsilon_L & \sqrt{2} \alpha_{(1)} \delta_L & 1 + 3\beta \left(\frac{v_L}{M_f} \right)^2 \end{pmatrix}_{LL}, \quad (17)$$

where

$$F = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \quad (18)$$

The large neutrino mixing angle between ν_μ and ν_τ indicated from the atmospheric neutrino oscillation is obtained [8] ⁶ if

$$\beta \left(\frac{v_L}{M_f} \right)^2 \ll \alpha_{(1)} \delta_L. \quad (19)$$

We also see large neutrino mixings between ν_e and $\nu_{\mu,\tau}$ from the mass matrices eq.(16) and eq.(17) for $\beta(v_L/M_f)^2 \leq \alpha_{(2)} \epsilon_L$. By using $\Delta m_{23}^2 (\equiv m_{\nu_3}^2 - m_{\nu_2}^2) \simeq 10^{-3} \text{eV}^2$ for

⁶The mixing matrix U which determines neutrino oscillations is defined as $U = U_\ell^\dagger U_\nu$, where U_ℓ and U_ν are diagonalization matrices for the charged lepton and neutrino mass matrices in eq.(16) and eq.(17), respectively (see ref.[8] for the definition of U_ℓ and U_ν).

the $\nu_\mu - \nu_\tau$ oscillation [5] (which corresponds to $m_{\nu_i} = \mathcal{O}(0.1)\text{eV}$ ⁷), $\delta_L \simeq 0.1$ and $\epsilon_L \simeq 10^{-3} - 10^{-2}$, we obtain⁸

$$\Delta m_{12}^2 \simeq \frac{\epsilon_L}{\delta_L} \Delta m_{23}^2 \simeq 10^{-5} - 10^{-4} \text{eV}^2, \quad (20)$$

for the $\nu_e - \nu_{\mu,\tau}$ oscillation. This is consistent with the large angle MSW solution [10] to the solar neutrino problem.⁹ For $\alpha_{(2)}\epsilon_L \ll \beta(v_L/M_f)^2 \ll a_{(1)}\delta_L$ we have small mixings between ν_e and $\nu_{\mu,\tau}$. However, we obtain, in this case, $\Delta m_{12}^2 \gg 10^{-5} - 10^{-4} \text{eV}^2$ which is too large for the small angle MSW solution [10]. Therefore, we consider that this is an unlikely case. From the above argument we may conclude that $(v_L/M_f) \leq \sqrt{\epsilon_L} \simeq 0.03 - 0.1$. This implies small $\tan \beta = \langle H \rangle / \langle \overline{H} \rangle \simeq \mathcal{O}(1)$ unless $\kappa_E(v_R/M_f)$ is very large.

We now turn to the quark sector, in which three doublet quarks $q_i (i = 1 - 3)$ transform as an $O(3)_L$ triplet while three down quarks $\overline{d}_i (i = 1 - 3)$ and the three up quarks $\overline{u}_i (i = 1 - 3)$ as $O(3)_R$ triplets. The mass matrix of down quarks is given in the hierarchical base by

$$F^T \widehat{m}_D F = \kappa_D \left(\frac{v_L v_R}{M_f^2} \right) \langle \overline{H} \rangle \begin{pmatrix} 2C_{22}^D \epsilon_L \epsilon_R & 2\sqrt{3}C_{21}^D \epsilon_R \delta_L & \sqrt{6}A_2^D \epsilon_R \\ 2\sqrt{3}C_{12}^D \epsilon_L \delta_R & 6C_{11}^D \delta_L \delta_R & 3\sqrt{2}A_1^D \delta_R \\ \sqrt{6}B_2^D \epsilon_L & 3\sqrt{2}B_1^D \delta_L & 3 \end{pmatrix}_{RL}. \quad (21)$$

For up quarks, the mass matrix is given by replacing D and $\langle \overline{H} \rangle$ with U and $\langle H \rangle$, respectively, in eq.(21). Quark mass eigenvalues are given by

$$\left(\frac{m_2^Q}{m_3^Q} \right) \simeq 2(A_1^Q B_1^Q - C_{11}^Q) \delta_L \delta_R \equiv X_2^Q \delta_L \delta_R, \quad (22)$$

and

$$\begin{aligned} \left(\frac{m_1^Q}{m_3^Q} \right) &= \frac{2}{3} (A_2^Q B_2^Q C_{11}^Q + A_1^Q B_1^Q C_{22}^Q - C_{11}^Q C_{22}^Q - A_2^Q B_1^Q C_{12}^Q - A_1^Q B_2^Q C_{21}^Q + C_{12}^Q C_{21}^Q) \epsilon_L \epsilon_R \\ &\equiv X_1^Q \epsilon_L \epsilon_R, \end{aligned} \quad (23)$$

⁷The constraint of m_{ν_i} is given by the new upper limit of the double β decay experiment $m_{eff}^{ee} < 0.2 \text{eV}$ [6]. The present model is viable without any accidental cancellation [9].

⁸If we use $\epsilon_L/\delta_L \simeq 0.1$ suggested from quark mixings, we get $\Delta m_{12}^2 \simeq 10^{-4} \text{eV}^2$. We consider that this is still consistent with the large MSW solution taking account of $\mathcal{O}(1)$ ambiguity.

⁹The current data [11] of Super-Kamiokande experiments give $\Delta m_{12}^2 \simeq 2 \times 10^{-5} - 2 \times 10^{-4} \text{eV}^2$ and $\sin^2 2\theta_{12} = 0.60 - 0.97$ at the 99% confidence level, for the large MSW solution.

where $Q = D$ or U . We assume that the parameters A_i^Q, B_i^Q, C_{ij}^Q are of $\mathcal{O}(1)$ as in the case of the lepton sector. We must, however, take $\kappa_U(v_R/M_f) \simeq \mathcal{O}(10)$ for $(v_L/M_f) \simeq 0.03$ to obtain the large mass of the top quark.

The CKM mixing angles are given by

$$\begin{aligned} |V_{us}| &\simeq s_{12}^D - s_{12}^U, \\ |V_{cb}| &\simeq s_{23}^D - s_{23}^U, \\ |V_{ub}| &\simeq s_{13}^D - s_{12}^U s_{23}^D + s_{12}^U s_{23}^U - s_{13}^U, \end{aligned} \quad (24)$$

where s_{ij}^Q denotes $\sin \theta_{ij}^Q$ and

$$\begin{aligned} s_{12}^Q &\simeq \frac{1}{\sqrt{3}} \frac{B_1^Q B_2^Q + 2C_{11}^Q C_{12}^Q \delta_R^2}{(B_1^Q)^2 + 2(C_{11}^Q)^2 \delta_R^2} \frac{\epsilon_L}{\delta_L} \equiv \frac{1}{\sqrt{3}} Y_{12}^Q \frac{\epsilon_L}{\delta_L}, \\ s_{23}^Q &\simeq \frac{\sqrt{2} B_1^Q}{1 + 2(A_1^Q)^2 \delta_R^2} \delta_L \equiv \sqrt{2} Y_{23}^Q \delta_L, \\ s_{13}^Q &\simeq -\sqrt{\frac{2}{3}} \frac{B_2^Q + 2A_1^Q C_{12}^Q \delta_R^2}{1 + 2(A_1^Q)^2 \delta_R^2} \epsilon_L \equiv -\sqrt{\frac{2}{3}} Y_{13}^Q \epsilon_L. \end{aligned} \quad (25)$$

Putting the experimental quark mass ratios and CKM matrix elements:

$$\frac{m_d}{m_b} \simeq \lambda^4, \quad \frac{m_s}{m_b} \simeq \lambda^2, \quad |V_{us}| \simeq \lambda, \quad |V_{cb}| \simeq \lambda^2, \quad (26)$$

we obtain the order of parameters as follows:

$$\delta_L \simeq \lambda^2, \quad \delta_R \simeq 1, \quad \epsilon_L \simeq \lambda^3, \quad \epsilon_R \simeq \lambda, \quad (27)$$

with $\lambda \simeq 0.2$. Here, we have assumed that $X_1^D, X_2^D, Y_{12}^D, Y_{23}^D, Y_{12}^U$ and Y_{23}^U are of $\mathcal{O}(1)$.

¹⁰ Then, we predict $|V_{ub}| \simeq \epsilon_L \simeq \lambda^3$, which is consistent with the experimental value [12].

The magnitudes of $\delta_{L,R}$ and $\epsilon_{L,R}$ in eq.(27) are what we have taken in the discussion on the lepton sector. Thus our model is successful to explain both lepton and quark mass matrices.

¹⁰To obtain the larger mass hierarchy $m_u/m_t \simeq \lambda^8$ and $m_c/m_t \simeq \lambda^4$ in the up-quark sector we must assume cancellation among $\mathcal{O}(1)$ parameters in X_1^U, X_2^U . A way of avoiding this fine-tuning is to introduce a new $O(3)'_R$, under which up quarks \bar{u}_i transform as a triplet. In this case the up quark mass matrix depends on breaking parameters of the additional $O(3)'_R$.

We have considered, in this letter, a model where ℓ_i and q_i belong to triplets of one $O(3)$ and \bar{e}_i , \bar{d}_i and \bar{u}_i belong to triplets of the other $O(3)$. We should note here that there is another interesting assignment that ℓ_i and \bar{d}_i are triplets of the $O(3)_L$ while \bar{e}_i , q_i and \bar{u}_i transform as triplets of the $O(3)_R$. At a first glance this model does not seem to work well, since the up quarks have $O(3)_R$ -invariant degenerate masses as the neutrinos. However, this problem may be easily solved by imposing a discrete symmetry such as Z_6 . The Z_6 charges of relevant fields are shown in Table 1 together with $O(3)_L \times O(3)_R$ representations. With this Z_6 we obtain the "democratic" mass matrices for charged leptons, down quarks and up quarks, and the neutrinos have almost degenerate mass as in the previous model. Remarkable point in this model is that the mass hierarchy in the up quark sector is explained by taking a hierarchy $\delta_R \simeq 0.1$ and $\epsilon_R \simeq 0.01$. The milder mass hierarchies in charged lepton and down quark sectors are obtained by taking $\delta_L \simeq 1$ and $\epsilon_L \simeq 0.1$. The CKM matrix is determined by $O(3)_R$ breaking parameters δ_R and ϵ_R , which turns out to be also consistent with the observations taking account of $\mathcal{O}(1)$ ambiguity. As for the neutrino mass matrix we obtain almost the same as before, except that the masses m_{ν_i} are reduced to $m_{\nu_i} \simeq \mathcal{O}(0.03)\text{eV}$ due to the larger value of δ_L .

In this letter we have assumed specific forms of $\Sigma_{L,R}^{(i)}$ in eq.(1) and eq.(2). However, even if $\Sigma_{L,R}^{(i)}$ have general forms, $(\Sigma_{L,R}^{(1)})_{ij} = (a)_{ij}\delta_{L,R}$ and $(\Sigma_{L,R}^{(2)})_{ij} = (b)_{ij}\epsilon_{L,R}$, our main conclusion does not change as long as ϵ_L , $\delta_L \ll 1$ and ϵ_R , $\delta_R \leq 1$. With the general form of $\Sigma_{L,R}^{(i)}$ we have vacuum-expectation values of ϕ_L and ϕ_R as

$$\langle \phi_L \rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix} v_L, \quad \langle \phi_R \rangle = \begin{pmatrix} a' \\ b' \\ c' \end{pmatrix} v_R, \quad (28)$$

with $a^2 + b^2 + c^2 = a'^2 + b'^2 + c'^2 = 3$. First of all, mass hierarchies in the charged lepton and quark sectors, and the small CKM mixing angles are explained by the hierarchy $\epsilon_L \ll \delta_L \ll 1$. On the other hand, we see that neutrino mixings are very large as long as vacuum-expectation values a , b and c in eq.(28) are of $\mathcal{O}(1)$. Therefore, the large neutrino mixings are generic predictions in the present mechanism of $O(3)_L \times O(3)_R$ breaking.

Finally, we should note that $\Sigma_{L,R}^{(i)}$ can be regarded as vacuum-expectation values of dynamical $\Sigma_{L,R}^{(i)}$ fields inducing the spontaneous $O(3)_L \times O(3)_R$ breaking. In this case we have six massless Nambu-Goldstone multiplets. However, the breaking scale $F_{L,R}$ are stringently constrained from various experimental results as $F_{L,R} > 10^9 \text{GeV}$ [13]. On the contrary, the flavor scale M_f may be as low as 10TeV in the case of explicit breaking.

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	ℓ_i	\bar{e}_i	q_i	\bar{d}_i	\bar{u}_i	ϕ_L	ϕ_R	H, \bar{H}	$\sigma_L^{(1),(2)}$	$\sigma_R^{(1),(2)}$
$O(3)_L$	3	1	1	3	1	3	1	1	5	1
$O(3)_R$	1	3	3	1	3	1	3	1	1	5
Z_6	1	1	1	1	1	1	1	2	0	0

Table 1: Representations of relevant fields in $O(3)_L \times O(3)_R$ and their Z_6 charges. The operators $\bar{u}qH$ and $\ell(\phi_L\phi_L)\ell H^2$ are suppressed by this Z_6 symmetry.